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# Variable center of rotation concerning the physiological motion of the knee theorem

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## Declaration

Given the natural flexion/extension movement of the knee as a combination of rolling and sliding during the first 30° of flexion; subsequently the real flexion depends upon the position of the center of rotation.

This implies that there is a difference between the knee angle measured with a goniometric sytem and the real distance between the foot and the axis of rotation.

## Hypothesis

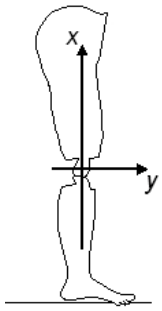
After an initial circular rotating phase, the flexion/extension movement of the knee proceeds with a combination of rolling and sliding (rotary - traslational) motion, which is characterized by the progressive decrease of the distance between the starting rotation and the articular surface.

## Thesis

The knee flexion/extension axis achieved after a rolling and sliding motion, is different from the axis measured only with a protractor; thus a correct evaluation of the flexion/extension motion of the knee should consider the instantaneous displacement of center of rotation.

## Proof

The position of the instantaneous center of rotation of the knee is initially placed on the x axis as an extension of the longitudinal axis of the thigh, whereas during the full extension of the thigh, it overlaps the longitudinal axis x of the leg itself. In this case the initial center of rotation of the knee coincides with the origin of Cartesian coordinate system xOy.




Nota 1

Provided that $R_a$ is the first rotating radius that lies on the y axis, we could describe the gradual displacement of the axis of rotation of the knee, the trajectory of flexion and extension of the leg on the thigh can be analitically defined as:

according to **α** <30 °, the knee motion can be described as a rigid system that rotates around a fixed center: the trajectory performed by the point P is the equation of circumference

$$\mathbf{x^2+y^2=R_a^2.}$$

According to 30° ≤ α < 135°, the center of rotation moves toward the articular surface by an amount equal to $\boldsymbol{\Delta_x}$.

The new coordinates of point P become:

$$\mathbf{X = x + \Delta_x}$$

$$\mathbf{Y = y + \Delta_y}$$

The equation of the center of rotation of the knee appears to be:

$$\mathbf{X^2 + Y^2 = R_b^2}$$

when $R_b$ is the real (effective) radius of rotation changing with the mutation of **α**, when $R_b < R_a$

When **α** is the angle between the x-axis and the radius of rotation, the values of x, X, y and Y can be obtained as:

$$x = R_a \text{ sen } \boldsymbol{\alpha}$$

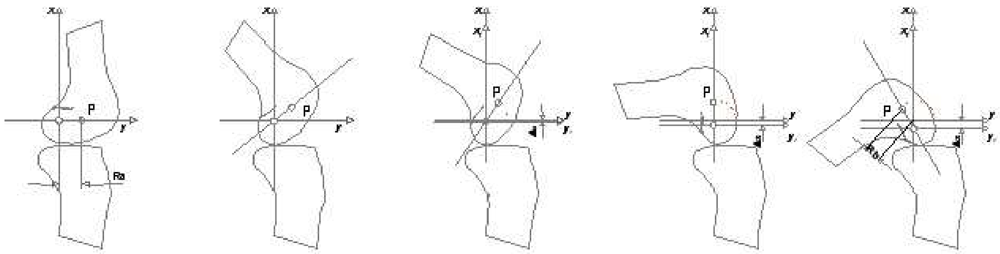
$$y = R_a \cos \boldsymbol{\alpha}$$

$$X = R_b \text{ sen } \boldsymbol{\alpha}$$

$$Y = R_b \cos \boldsymbol{\alpha}$$

So when a given value of **α** is included between 30° and 135° the position of the istantaneous center of rotation can be calculated as:

$$\Delta_x = X - x = (R_a - R_b) \text{ sen } \boldsymbol{\alpha}$$



**Pratical application – The new scale angle**

As seen from above, it appears that the rolling and sliding motion the peripheral point P follows a curve with a spiral process, receding toward the center.

This implies that the scale for the evaluation of the actual angle of the knee must be established by the sequence of extreme points P of radius $R_a$, which second end (it initially overlaps with the origin of the reference xOy system) after 30° it moves along the x axis within a degree equal to $\boldsymbol{\Delta_x}$

Therefore the angular reference information on the spiral curve (Fig. 1) must be placed according to the new source xO'Y as a result of the instantaneus shift of the goniometric system along the x axis, when O'= O + $\boldsymbol{\Delta_x}$, as a new instantaneus center of rotation.

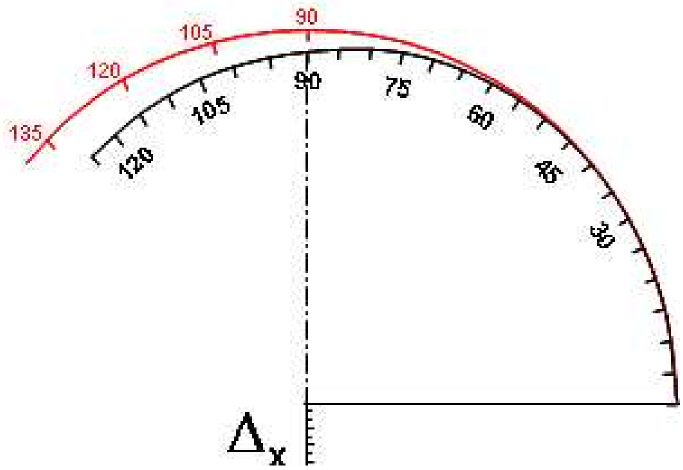



Fig.1

*Fig. 1 – Angular scale concerning $\boldsymbol{R_a}$= 13mm and $\boldsymbol{\Delta_x}$= 3,6mm*[1]

**Reference**

1) Pellis G., Di Cosmo F.: Il moto roto-traslatorio: Studio sperimentale, analisi matematica, dispositivo ortopedico, Calzetti Editore, Perugia, Atti del XVIII edizione del Convegno di Traumatologia e Riabilitazione Sportiva, Bologna, 2009.